

Absorption of fixed scalar in scattering off 4D N=4 black holes

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Abstract

We perform the perturbation analysis of the black holes in the 4D, N=4 supergravity. Analysis around the black holes reveals a complicated mixing between the dilaton and other fields (metric and two U(1) Maxwell fields). It turns out that considering both s-wave ($l = 0$) and higher momentum modes ($l \neq 0$), the dilaton as a fixed scalar is the only propagating mode with $P = Q, h_1 = h_2 = 0$ and $\mathcal{F} = -\mathcal{G} = 2\varphi$. We calculate the absorption cross-section for scattering of low frequency waves of fixed scalar and U(1) Maxwell fields off the extremal black hole.

Recently the absorption of fixed scalars into 4D extremal black holes was considered in [1,2]. This is found to be suppressed compared with that of the free scalar. Actually the low energy absorption cross section for minimally coupled scalars turned out to be proportional to the area of the black hole horizon [3,4]. On the other hand, the fixed scalar cross section, as opposed to the Hawking decay, was found to vanish as ω^2 in the classical limit when $\omega \rightarrow 0$. The authors in [1,2] used only part of the perturbing equations to decouple the dilaton from other fields (metric and two U(1) Maxwell fluctuations). However one has to consider all perturbing equations around the black holes to find the consistent solution [5].

Holzhey and Wilczek have studied thoroughly the perturbations of the dilaton black hole with one U(1) charge [6]. They started with fourteen equations. However there exist only five (two metric, two U(1) Maxwell, and one dilaton) physical degrees of freedom. The equations governing the perturbation of metric, U(1) Maxwell, and dilaton fields can be reduced to five wave equations corresponding to five independent modes. Actually these modes consist of various linear combinations of original functions parametrizing the perturbations and thus their direct physical meaning is not transparent. We note that it is almost impossible to decouple the dilaton from other fields. Further the corresponding potentials are too unwieldy to allow a useful description in closed form and analytical analysis. Fortunately they found that the combined modes show the same qualitative behavior as the minimally coupled scalar. Since the field equation for a free scalar is remarkably simple, many authors consider it as a spectator rather than real physical field in studying the black holes.

In this letter, we shall perform a complete analysis of the perturbation for the 4D, N=4 black holes with two U(1) charges [7]. This model with two U(1) charges provides us a prototype for obtaining the absorption cross-section of the decoupled dilaton (as a fixed scalar). Also this analysis will be a cornerstone for testing effective string models of the 5D black hole with fixed scalars [8,9]. The calculations of absorption and emission rates for relevant fields by the extremal and near-extremal black holes are very important to compare them with the results of D-branes. Apart from the counting of states [10], nowadays this is an important issue [11].

We start with the bosonic sector of the 4D, N=4 supergravity [7]

$$S = \int d^4x \sqrt{-g} \left\{ R - 2(\nabla\phi)^2 - e^{-2\phi} F^2 - e^{2\phi} G^2 \right\} \quad (1)$$

with the MTW conventions [12]. Here the fields are metric $g_{\mu\nu}$, dilaton ϕ , Maxwell fields $F_{\mu\nu}$, and $G_{\mu\nu}$. When $G_{\mu\nu}$ is absent, the above model reduces to the dilaton black hole with $a = 1$ in [6]. The equations of motion are given by

$$R_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi - 2e^{-2\phi}F_{\mu\rho}F_\nu{}^\rho + \frac{1}{2}e^{-2\phi}F^2g_{\mu\nu} - 2e^{2\phi}G_{\mu\rho}G_\nu{}^\rho + \frac{1}{2}e^{2\phi}G^2g_{\mu\nu} = 0, \quad (2)$$

$$\nabla^2\phi + \frac{1}{2}e^{-2\phi}F^2 - \frac{1}{2}e^{2\phi}G^2 = 0, \quad (3)$$

$$\nabla_\mu F^{\mu\nu} - 2(\partial_\mu\phi)F^{\mu\nu} = 0, \quad (4)$$

$$\nabla_\mu G^{\mu\nu} + 2(\partial_\mu\phi)G^{\mu\nu} = 0. \quad (5)$$

The static charged black hole solution to the above equations is given by the background metric

$$ds^2 = -(H_1 H_2)^{-1} dt^2 + H_1 H_2 (dr^2 + r^2 d\Omega_2^2) \quad (6)$$

and

$$e^{2\bar{\phi}} = \frac{H_2}{H_1}, \quad \bar{F} = \frac{1}{\sqrt{2}} dH_1^{-1} \wedge dt, \quad \bar{G} = \frac{1}{\sqrt{2}} dH_2^{-1} \wedge dt \quad (7)$$

with two harmonic functions

$$H_1 = 1 + \frac{\sqrt{2}Q}{r}, \quad H_2 = 1 + \frac{\sqrt{2}P}{r}. \quad (8)$$

Here Q and P are two U(1) charges. We note that the event horizon is located at $r_{EH} = 0$. The fixed scalars are defined as the special massless fields whose values on the horizon of the extremal black hole are fixed by the U(1) charges. Considering (7), $\lim_{r \rightarrow 0} e^{2\bar{\phi}} = P/Q$. In the extremal limit this ratio is one and thus the dilaton can be regarded as a fixed scalar. Similarly, we have $\lim_{r \rightarrow 0} \bar{F}^2 = -1/2Q^2$ and $\lim_{r \rightarrow 0} \bar{G}^2 = -1/2P^2$. Therefore these may be considered as the fixed tensors. Before we proceed, let us count the degrees of freedom. We have two graviton, four U(1) Maxwell, and one dilaton degrees of freedom in four dimensions.

Here for simplicity we start with five among seven perturbation fields around the black hole background as [13]

$$F_{tr} = \bar{F}_{tr} + \mathcal{F}_{tr} = \bar{F}_{tr}[1 + \mathcal{F}(t, r, \theta, \phi)], \quad (9)$$

$$G_{tr} = \bar{G}_{tr} + \mathcal{G}_{tr} = \bar{G}_{tr}[1 + \mathcal{G}(t, r, \theta, \phi)], \quad (10)$$

$$\phi = \bar{\phi} + \varphi(t, r, \theta, \phi), \quad (11)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (12)$$

Considering the gauge transformation $(h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu)$, we can make $h_{\mu\nu}$ into the diagonal form and further set the angular part of $h_{\mu\nu}$ to be zero. Then the metric perturbation is given by two small fields $h_1(t, r, \theta, \phi)$ and $h_2(t, r, \theta, \phi)$ [2]

$$h_{\mu\nu} = \text{diag}(-(H_1 H_2)^{-1} h_1, H_1 H_2 h_2, 0, 0). \quad (13)$$

One has to linearize (2)-(5) in order to obtain the equations governing the perturbations as

$$\begin{aligned} & \delta R_{\mu\nu}(h) - 2(\partial_\mu \bar{\phi} \partial_\nu \varphi + \partial_\mu \varphi \partial_\nu \bar{\phi}) + 2e^{-2\bar{\phi}}(-2\bar{F}_{\mu\rho} \mathcal{F}_\nu{}^\rho + \bar{F}_{\mu\rho} \bar{F}_{\nu\alpha} h^{\rho\alpha} + 2\bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho \varphi) \\ & + e^{-2\bar{\phi}} \bar{g}_{\mu\nu} (\bar{F}_{\sigma\rho} \mathcal{F}^{\sigma\rho} - \bar{F}_{\rho\sigma} \bar{F}_\alpha{}^\sigma h^{\rho\alpha} - \bar{F}^2 \varphi) + \frac{1}{2} e^{-2\bar{\phi}} \bar{F}^2 h_{\mu\nu} \\ & + 2e^{2\bar{\phi}} (-2\bar{G}_{\mu\rho} \mathcal{G}_\nu{}^\rho + \bar{G}_{\mu\rho} \bar{G}_{\nu\alpha} h^{\rho\alpha} - 2\bar{G}_{\mu\rho} \bar{G}_\nu{}^\rho \varphi) \\ & + e^{2\bar{\phi}} \bar{g}_{\mu\nu} (\bar{G}_{\sigma\rho} \mathcal{G}^{\sigma\rho} - \bar{G}_{\rho\sigma} \bar{G}_\alpha{}^\sigma h^{\rho\alpha} + \bar{G}^2 \varphi) + \frac{1}{2} e^{-2\bar{\phi}} \bar{G}^2 h_{\mu\nu} = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} & \bar{\nabla}^2 \varphi - h^{\mu\nu} \bar{\nabla}_\mu \bar{\nabla}_\nu \bar{\phi} - \bar{g}^{\mu\nu} \delta \Gamma_{\mu\nu}^\rho(h) \partial_\rho \bar{\phi} + e^{-2\bar{\phi}} \bar{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - e^{-2\bar{\phi}} \bar{F}_{\mu\nu} \bar{F}_\rho{}^\nu h^{\mu\rho} - e^{-2\bar{\phi}} \bar{F}^2 \varphi \\ & - e^{2\bar{\phi}} \bar{G}_{\mu\nu} \mathcal{G}^{\mu\nu} + e^{2\bar{\phi}} \bar{G}_{\mu\nu} \bar{G}_\rho{}^\nu h^{\mu\rho} - e^{2\bar{\phi}} \bar{G}^2 \varphi = 0, \end{aligned} \quad (15)$$

$$(\bar{\nabla}_\mu - 2\partial_\mu \bar{\phi})(\mathcal{F}^{\mu\nu} - \bar{F}_\alpha{}^\nu h^{\alpha\mu} - \bar{F}^\mu{}_\beta h^{\beta\nu}) + \bar{F}^{\mu\nu} (\delta \Gamma_{\sigma\mu}^\sigma(h) - 2(\partial_\mu \varphi)) = 0, \quad (16)$$

$$(\bar{\nabla}_\mu + 2\partial_\mu \bar{\phi})(\mathcal{G}^{\mu\nu} - \bar{G}_\alpha{}^\nu h^{\alpha\mu} - \bar{G}^\mu{}_\beta h^{\beta\nu}) + \bar{G}^{\mu\nu} (\delta \Gamma_{\sigma\mu}^\sigma(h) + 2(\partial_\mu \varphi)) = 0, \quad (17)$$

where

$$\delta R_{\mu\nu}(h) = -\frac{1}{2}\bar{\nabla}_\mu\bar{\nabla}_\nu h^\rho{}_\rho - \frac{1}{2}\bar{\nabla}^\rho\bar{\nabla}_\rho h_{\mu\nu} + \frac{1}{2}\bar{\nabla}^\rho\bar{\nabla}_\nu h_{\rho\mu} + \frac{1}{2}\bar{\nabla}^\rho\bar{\nabla}_\mu h_{\nu\rho}, \quad (18)$$

$$\delta\Gamma_{\mu\nu}^\rho(h) = \frac{1}{2}\bar{g}^{\rho\sigma}(\bar{\nabla}_\nu h_{\mu\sigma} + \bar{\nabla}_\mu h_{\nu\sigma} - \bar{\nabla}_\sigma h_{\mu\nu}). \quad (19)$$

Solving (16) and (17), one can express two U(1) scalars (\mathcal{F} and \mathcal{G}) in terms of φ, h_1, h_2 as

$$2\mathcal{F} = h_1 + h_2 + 4\varphi, \quad 2\mathcal{G} = h_1 + h_2 - 4\varphi. \quad (20)$$

This means that on-shell, \mathcal{F} and \mathcal{G} are no longer the independent modes. Also from (14) six off-diagonal elements are given by

$$(t, r) : \quad \frac{1}{2} \left[\left(\frac{H'_1}{H_1} + \frac{H'_2}{H_2} \right) + \frac{2}{r} \right] \partial_t h_2 + \left(\frac{H'_1}{H_1} - \frac{H'_2}{H_2} \right) \partial_t \varphi = 0, \quad (21)$$

$$(t, \theta) : \quad \partial_t \partial_\theta h_2 = 0, \quad (22)$$

$$(t, \phi) : \quad \partial_t \partial_\phi h_2 = 0, \quad (23)$$

$$(r, \theta) : \quad \frac{1}{2}(\partial_r - \Gamma_{r\theta}^\theta) \partial_\theta h_1 + \frac{1}{2r} \partial_\theta (h_1 - h_2) - \left(\frac{H'_1}{H_1} - \frac{H'_2}{H_2} \right) \partial_\theta \varphi = 0, \quad (24)$$

$$(r, \phi) : \quad \frac{1}{2}(\partial_r - \Gamma_{r\phi}^\phi) \partial_\phi h_1 + \frac{1}{2r} \partial_\phi (h_1 - h_2) - \left(\frac{H'_1}{H_1} - \frac{H'_2}{H_2} \right) \partial_\phi \varphi = 0, \quad (25)$$

$$(\theta, \phi) : \quad \frac{1}{2}(\partial_\theta - \Gamma_{\theta\phi}^\phi) \partial_\phi (h_1 + h_2) = 0, \quad (26)$$

where the prime (\prime) means the differentiation with respect to r . And four diagonal elements of (14) take the form

$$\begin{aligned} (t, t) : \quad & - (H_1 H_2)^2 \partial_t^2 h_2 + (\partial_r^2 + \frac{2}{r} \partial_r) h_1 + \frac{1}{r^2} (\partial_\theta^2 + \cot \theta \partial_\theta + \frac{1}{\sin^2 \theta} \partial_\phi^2) h_1 \\ & - \left(\frac{H'_2}{H_2} + \frac{H'_1}{H_1} \right) \frac{(h'_1 - h'_2)}{2} + \left(\frac{H_1'^2}{H_1^2} + \frac{H_2'^2}{H_2^2} \right) (h_1 - h_2) \\ & + \frac{2Q^2}{r^4 H_1^2} (h_2 + 2\varphi - 2\mathcal{F}) + \frac{2P^2}{r^4 H_2^2} (h_2 - 2\varphi - 2\mathcal{G}) = 0, \end{aligned} \quad (27)$$

$$\begin{aligned} (r, r) : \quad & - (H_1 H_2)^2 \partial_t^2 h_2 + \partial_r^2 h_1 - \frac{2}{r} \partial_r h_2 + \frac{1}{r^2} (\partial_\theta^2 + \cot \theta \partial_\theta + \frac{1}{\sin^2 \theta} \partial_\phi^2) h_2 + \\ & - 4 \left(\frac{H'_1}{H_1} - \frac{H'_2}{H_2} \right) \varphi' - \left(\frac{H'_1}{H_1} + \frac{H'_2}{H_2} \right) \frac{(3h'_1 + h'_2)}{2} \\ & + \frac{2Q^2}{r^4 H_1^2} (h_1 + 2\varphi - 2\mathcal{F}) + \frac{2P^2}{r^4 H_2^2} (h_1 - 2\varphi - 2\mathcal{G}) = 0, \end{aligned} \quad (28)$$

$$\begin{aligned} (\theta, \theta) : \quad & \frac{1}{r^2} \partial_\theta^2 (h_1 + h_2) - \frac{2}{r^2} h_2 + \left(\frac{H_1'^2}{H_1^2} + \frac{H_2'^2}{H_2^2} \right) h_2 + \left[\left(\frac{H'_1}{H_1} + \frac{H'_2}{H_2} \right) + \frac{2}{r} \right] \frac{(h'_1 - h'_2)}{2} \\ & - \frac{2Q^2}{r^4 H_1^2} (h_1 + h_2 + 2\varphi - 2\mathcal{F}) - \frac{2P^2}{r^4 H_2^2} (h_1 + h_2 - 2\varphi - 2\mathcal{G}) = 0, \end{aligned} \quad (29)$$

$$\begin{aligned}
(\phi, \phi) : \quad & \frac{1}{r^2 \sin^2 \theta} \partial_\phi^2 (h_1 + h_2) - \frac{2}{r^2} h_2 + \left(\frac{H_1'^2}{H_1^2} + \frac{H_2'^2}{H_2^2} \right) h_2 \\
& + \frac{\cot \theta}{r^2} \partial_\theta (h_1 + h_2) + \left[\left(\frac{H_1'}{H_1} + \frac{H_2'}{H_2} \right) + \frac{2}{r} \right] \frac{h_1' - h_2'}{2} \\
& - \frac{2Q^2}{r^4 H_1^2} (h_1 + h_2 + 2\varphi - 2\mathcal{F}) - \frac{2P^2}{r^4 H_2^2} (h_1 + h_2 - 2\varphi - 2\mathcal{G}) = 0.
\end{aligned} \tag{30}$$

The dilaton equation (15) leads to

$$\begin{aligned}
& -(H_1 H_2)^2 \partial_t^2 \varphi + \varphi'' + \frac{2}{r} \varphi' + \frac{1}{r^2} (\partial_\theta^2 + \cot \theta \partial_\theta + \frac{1}{\sin^2 \theta} \partial_\phi^2) \varphi \\
& + \left(\frac{H_2'^2}{H_2^2} - \frac{H_1'^2}{H_1^2} \right) \frac{h_2}{2} + \left(\frac{H_2'}{H_2} - \frac{H_1'}{H_1} \right) \frac{h_1' - h_2'}{4} \\
& + \frac{Q^2}{r^4 H_1^2} (h_1 + h_2 + 2\varphi - 2\mathcal{F}) - \frac{P^2}{r^4 H_2^2} (h_1 + h_2 - 2\varphi - 2\mathcal{G}) = 0.
\end{aligned} \tag{31}$$

Let us first consider the s-wave ($l = 0$ partial wave). This implies that the fluctuations are functions of t and r only. In this case the relevant relation comes only from (21) among six off-diagonal elements. By integrating (21) over time, we can obtain one relation

$$\left[\left(\frac{H_1'}{H_1} + \frac{H_2'}{H_2} \right) + \frac{2}{r} \right] h_2 = -2 \left(\frac{H_1'}{H_1} - \frac{H_2'}{H_2} \right) \varphi. \tag{32}$$

Adding (θ, θ) and (ϕ, ϕ) component equations leads to the other relation

$$\left[\left(\frac{H_1'}{H_1} + \frac{H_2'}{H_2} \right) + \frac{2}{r} \right] (h_1' - h_2') = -2 \left[\left(\frac{H_1'^2}{H_1^2} + \frac{H_2'^2}{H_2^2} \right) - \frac{2}{r^2} \right] h_2 - 4 \left(\frac{H_1'^2}{H_1^2} - \frac{H_2'^2}{H_2^2} \right) \varphi. \tag{33}$$

Note that (32) and (33) are valid only for different metric perturbations ($h_1 \neq h_2$) and $l = 0$ case. Also one finds the relation,

$$\left[\left(\frac{H_1'}{H_1} + \frac{H_2'}{H_2} \right) + \frac{2}{r} \right] (h_1' + h_2') = -4 \left(\frac{H_1'}{H_1} - \frac{H_2'}{H_2} \right) \varphi', \tag{34}$$

by subtracting (28) from (27). And subtracting (34) from (33) leads to

$$\begin{aligned}
\left[\left(\frac{H_1'}{H_1} + \frac{H_2'}{H_2} \right) + \frac{2}{r} \right] h_2' &= -2 \left(\frac{H_1'}{H_1} - \frac{H_2'}{H_2} \right) \varphi' \\
&+ \left[\left(\frac{H_1'^2}{H_1^2} + \frac{H_2'^2}{H_2^2} \right) - \frac{2}{r^2} \right] h_2 + 2 \left(\frac{H_1'^2}{H_1^2} - \frac{H_2'^2}{H_2^2} \right) \varphi.
\end{aligned} \tag{35}$$

But (35) turns out to be redundant because it can be derived by differentiation of (32).

Only for s-wave perturbation, we can obtain the dilaton equation by substituting (32) and (33) into (31):

$$\left[r^{-2} \partial_r r^2 \partial_r - (H_1 H_2)^2 \partial_t^2 - \frac{4(P+Q)^2}{r^2(\sqrt{2}P + \sqrt{2}Q + 2r)^2} \right] \varphi = 0, \quad (36)$$

which is exactly the same as (13) of Ref. [2]. However, we have to consider both $l = 0$ and $l \neq 0$ modes [14] equations to obtain the correct dilaton equation.

Considering higher angular momentum modes ($l \neq 0$), all equations (21)–(30) are satisfied simultaneously when

$$P = Q, \quad h_1 = h_2 = 0, \quad \mathcal{F} = -\mathcal{G}. \quad (37)$$

In order to obtain the radial equation, we separate the dilaton as

$$\varphi(t, r, \theta, \phi)_{lm} = e^{-i\omega t} \varphi(r) Y_{lm}(\theta, \phi). \quad (38)$$

Inserting (37) and (38) into (31), one finds the fixed scalar equation

$$\varphi'' + \frac{2}{r} \varphi' + \frac{(r+1)^4 \omega^2}{r^4} \varphi - \frac{l(l+1)}{r^2} \varphi - \frac{2}{r^2(r+1)^2} \varphi = 0, \quad (39)$$

where the mass of the black hole is chosen as $M = \sqrt{2}Q = \sqrt{2}P = 1$. Considering (20) and (37), the equations for U(1) fluctuations (\mathcal{F}, \mathcal{G}) are identical to the fixed scalar equation (39). If the last term in (39) is absent, it corresponds to the free scalar. Here we wish to distinguish (39) from (36). Eq.(36) is consistent with (39) when $P = Q$ and $l = 0$. The authors of Ref. [2] used only the s-wave fluctuation equations to decouple the dilaton from the other fields and thus found the equation (36). Actually we consider both $l = 0$ and $l \neq 0$ modes to obtain (39). Also the U(1) fluctuations are not considered in [2].

Further, we can recover Kol and Rajaraman's result [1] if we transform the location of the event horizon from $r_{EH} = 0$ to $r'_{EH} = 1$ by $r \rightarrow r' - 1$. And then neglecting the prime, we have the new metric element

$$ds^2 = -\frac{(r-1)^2}{r^2} dt^2 + \frac{r^2}{(r-1)^2} dr^2 + r^2 d\Omega^2. \quad (40)$$

Here we note that this extremal black hole do have the non-zero area ($A_H = 4\pi$). One finds the s-wave equation from (39),

$$\varphi'' + \frac{2}{r-1}\varphi' + \frac{r^4\omega^2}{(r-1)^4}\varphi - \frac{2}{r^2(r-1)^2}\varphi = 0, \quad (41)$$

which is the same form as (19) in Ref. [1]. The authors in [1] used only the $U(1)$ Maxwell fluctuations to decouple the dilaton. Eventually this case is reduced to our case, although they did not use the metric fluctuations. This is because one finds $h_1 = h_2 = 0$ in our results.

One way of understanding a black hole is to find out how it reacts to external perturbation (fixed scalar). We visualize the black hole as presenting an effective potential barrier (or well) to the on-coming waves [15]. Here let us derive the s-wave absorption cross-section for the scattering of dilaton (and Maxwell fields) off the extremal black hole according to Kol and Rajaraman [1]. This can first be done by solving the equation (41) in three regions: the near region ($1 < r < 1 + \omega/\sqrt{2}$), the intermediate region ($1 + \omega/\sqrt{2} < r < 2^{1/4}/\sqrt{\omega}$) and the far region ($r > 2^{1/4}/\sqrt{\omega}$). The intermediate region is necessary for matching the solutions. One must have the ingoing waves at the event horizon ($r \rightarrow 1, \zeta \rightarrow \infty$) as $e^{i(\zeta - \omega t)}$, where $\zeta = \omega r/(r-1)$. In the near region, one can thus express the solution in terms of Coulomb wave functions as $\varphi_{near} = iF_1(\zeta) + G_1(\zeta)$. In the intermediate region one has $\varphi_{inter} = A/z + Bz^2$ with $z = \zeta/\omega$. Also we have both the incident (C) and reflected (D) waves ($r\varphi_{far} = CF_0(\omega r) + DG_0(\omega r)$) in the far region. According to the matching conditions, one finds $B/A = i\omega^3/3$ and $D/C = \omega(2B - A)/(A + B)$. Finally the absorption coefficient is given by the flux conservation ($\mathcal{A}(\omega) + \mathcal{R}(\omega) = 1$) as

$$\mathcal{A}(\omega) = 1 - \left| \frac{1 + i\frac{D}{C}}{1 - i\frac{D}{C}} \right|^2 = \frac{36\omega^4}{9 + 9\omega^2 + 18\omega^4 + \omega^6 + 4\omega^8}. \quad (42)$$

In the low frequency approximation of $\omega \ll 1$, the scattering of the dilaton off the extremal black hole is dominated by the $l = 0$ (s -wave) term. This leads to $\mathcal{A}(\omega) = 4\omega^4$. This means that the differential cross-section is independent of θ , just as it was in the classical case. The corresponding s -wave cross-section in four dimensions is then given by the optical theorem,

$$\sigma_s^{fixed} = \frac{\pi\mathcal{A}}{\omega^2} = 4\pi\omega^2. \quad (43)$$

On the other hand the free scalar cross-section is proportional to the area of the horizon as [4]

$$\sigma_s^{free} = A_H = 4\pi \quad (44)$$

for the low frequency scattering ($l = 0, \omega \rightarrow 0$). This cross-section is four times the geometric cross-section. This larger effective size is characteristic of long-wavelength scattering; in a sense, these waves feel their way around the whole sphere (S^2) on the event horizon.

In conclusion, we carried out a complete perturbation analysis of the black holes in the 4D, N=4 supergravity. It is shown that there is a complicated mixing between the dilaton and other fields (metric and U(1) Maxwell fields). Here we consider both $l = 0$ mode and $l \neq 0$ higher modes. Then the dilaton is decoupled from the metric and Maxwell fields when $P = Q, h_1 = h_2 = 0$ and $\mathcal{F} = -\mathcal{G} = 2\varphi$. The first means that one has a fixed scalar in the extremal limit. Also the second implies that we have no propagating gravitons in the extremal black hole. Finally the U(1) Maxwell perturbations are identical to that of the dilaton, and thus have the same absorption cross-section as in (43). This is found to vanish as (43) when $\omega \rightarrow 0$. The fixed scalars are defined as the special massless fields whose values on the horizon of the extremal black hole are fixed by the U(1) charges. In this sense two U(1) Maxwell fields belong to the fixed tensors. Here, these are reduced to the fixed scalars (\mathcal{F}, \mathcal{G}). Thus it is understood that the U(1) Maxwell fields have the same absorption cross-section as that of the dilaton in (43).

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